THE IMPLEMENTATION OF TECHNICAL APPLICATIONS TO THE TEACHING OF APPLIED MATHEMATICS

VAGASKÁ Alena – GOMBÁR Miroslav – KMEC Ján, SR

Abstract

The possibilities of the implementation of technical applications as well as ICT to the teaching of Applied Mathematics are presented in the article with the aim to contribute to the effectiveness of technical education at the faculties of technical universities.

Key words: technical education, applied mathematics, mathematical modelling, numerical methods, ICT.

IMPLEMENTÁCIA TECHNICKÝCH APLIKÁCIÍ VO VYUČOVANÍ APLIKOVANEJ MATEMATIKY

Resumé

V článku sú prezentované možnosti implementácie technických aplikácii ako aj využitie IKT vo vyučovaní Aplikovanej matematiky s cieľom prispieť k zvýšeniu efektívnosti technického vzdelávania na fakultách technických univerzít.

Kľúčové slová: technické vzdelávanie, aplikovaná matematika, matematické modelovanie, numerické metódy, IKT.

Introduction

At the faculties of technical universities, teaching of mathematics should be aimed at the development of students' abilities to apply their knowledge of mathematics to the technology, mathematical modeling as well as numerical solutions of engineering problems. That is why we appreciate the introduction of the subject Applied Mathematics at the Faculty of Manufacturing Technologies to the engineering studies. It provides us space to present a usefulness of mathematics, for example through applicative tasks, numerical solution of problems preferably related to engineering practice, eventually statistical treatment of experimentally obtained data with a successive modeling of technological processes based on CAS programs utilization.

Based on the information sheet of the subject Applied Mathematics at the Faculty of Manufacturing Technologies, we can find out that lectures and practical classes are aimed at students acquiring theoretical and practical knowledge of selected numerical methods and statistics provided that they will be able to use it in algorithms design and application to the solution of specific tasks with PC support. Practical classes exclusively take place in computing classrooms which owing to a complexity of mathematical calculations in technical applications is inevitable and natural. We bring some views on the teaching of the mentioned subject based on our teaching experience gained within the so called practical classes in computing [1].

1 MS Excel in Applied Mathematics

When solving specific tasks at practical classes in Applied Mathematics, an appropriate computing equipment is used, at present it is mainly MS Excel. Excel offers an enormous number of possible functions, tools and options for use. Students' enthusiasm resulting from the possibility of using MS Excel to solve tasks was evident as early as they started solving tasks of introductory topics of Applied Mathematics (functions approximation, numerical methods of nonlinear equations solution, numerical calculation of integral, …). Students appreciated the possibility of graphical interpretation and they realized that many time- consuming calculations can be carried out by MS Excel instead of them.

Our aim at practical classes was that students can acquire algorithms of numerical methods as well as their use with a suitable computing support. We concentrated on students knowledge of basic formulas related to the given numerical methods and students ability to recognize their limitations, weak spots. Students should be able to explain when the given method diverges and why. That is why we used some examples to show students a divergence of selected methods with both unsatisfying conditions for convergence and due to numerical instability of a calculation. Sometimes identical tasks were solved by different numerical methods, so that students were able to compare the accuracy, a complexity of a solving procedure, or a convergence speed. For example, when solving nonlinear equations it was possible to observe these attributes with the use of different numerical methods – students had to acquire the bisection method, the iterative method, the Newton's method and the regula-falsi method.

Individual topics and included tasks for solving at practical classes were chosen in order to emphasize the utilization of applied mathematics and computing support in technical practice. Let us present now a specific example to show some advantages of a computer-aided teaching of Applied Mathematics.

2 The Method of the Least Squares

In technical practice, it is often required to approximate (substitute) the measured values by a function which would as good as possible express a dependence between measured values. The advantage of an approximation by the method of least squares (MLS) is that an approximation polynomial does not necessarily pass through all events (measured values), and that is a user who can determine a degree of a polynomial. This has a great importance considering a technical practice, where a higher number o values is measured. The following example illustrates our effort to elucidate problems of a technical practice to our students.

When working a material, an influence of some parameters on a resulting roughness of a worked surface appeared. The influence of a current $I[\mathrm{A}]$ on a roughness Ra was recorded according to the table 1, where a variable x represents a current $I[\mathrm{A}]$ and a variable y represents a roughness Ra. Using the method of least squares determine a linear hyperbolic dependence as well as indices of a function correlation $f(x)$ given by the table.

Table 1

x	0,2	0,3	0,5	0,5	0,7	0,8
y	15,6	15,4	15,1	15,2	14,9	14,9

When explaining theoretical essentials of the MLS at practical classes, we start with the derivation of a linear dependence of the MLS. With the functions approximation by the MLS we search for the function $y = f(x)$; i.e. for a linear dependence $y = a_1 x + a_0$ so that a sum of squares of deviations

$$S = \sum_{i=1}^{n} \left(f^*(x_i) - f(x_i) \right)^2 \qquad (1)$$

is minimal. At the same time, $f^*(x_i)$ represents a real function, in our case $y_i^* = f^*(x_i) = a_1 x + a_0$; $y_i = f(x_i)$ represents the measured values [2], [3]. According to (1) in a linear dependence for a sum of squares of deviations it is valid:

$$S = \sum_{i=1}^{n} (a_1 x_i + a_0 - y_i)^2 \qquad (2)$$

As this sum has to be minimal, we search for the minimum of the function (2). It is the function of two variables a_1, a_0 - the coefficients of a linear dependence. It means that we need to calculate partial derivations $\frac{\partial S}{\partial a_0}, \frac{\partial S}{\partial a_1}$. After a modification we obtain a system of linear equations (SLE)

$$\begin{aligned} a_0 \sum_{i=1}^{n} 1 + a_1 \sum_{i=1}^{n} x_i &= \sum_{i=1}^{n} y_i \\ a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2 &= \sum_{i=1}^{n} x_i y_i \end{aligned} \qquad (3)$$

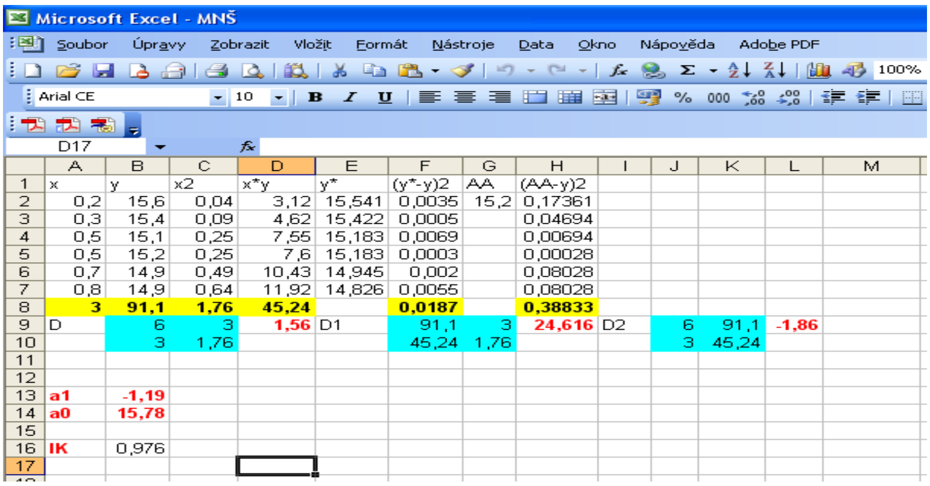
where $\sum_{i=1}^{n} 1 = n$, which is the number of the measured values. Through the solution of the SLE (3) we obtain a polynomial of the first degree $y = a_1 x + a_0$, i.e. a linear dependence of the MLS for the measured values.

When calculating the sums $\sum_{i=1}^{n} x_i$, $\sum_{i=1}^{n} x_i^2$, $\sum_{i=1}^{n} y_i$, $\sum_{i=1}^{n} x_i y_i$ it is suitable to use MS Excel. One of strong points of Microsoft Excel application is also a number of inbuilt functions, Excel saves our time using them and eliminating time-consuming calculations. According to (3), we can write down a system of linear equations for our example:

$$\begin{aligned} 6a_0 + 3a_1 &= 91{,}1 \\ 3a_0 + 1{,}76a_1 &= 45{,}24 \end{aligned} \qquad (4)$$

Based on the system (4), by the help of Cramer's rule we obtain $a_0 = \frac{D_1}{D}, a_1 = \frac{D_2}{D}$.

When calculating determinants D, D_1, D_2 we advantageously utilized in Excel inbuilt function DETERMINANT , see fig. 1.



	A	B	C	D	E	F	G	H	I	J	K	L	M
1	x	y	x2	x*y	y*	(y*-y)2	AA	(AA-y)2					
2	0,2	15,6	0,04	3,12	15,541	0,0035	15,2	0,17361					
3	0,3	15,4	0,09	4,62	15,422	0,0005		0,04694					
4	0,5	15,1	0,25	7,55	15,183	0,0069		0,00694					
5	0,5	15,2	0,25	7,6	15,183	0,0003		0,00028					
6	0,7	14,9	0,49	10,43	14,945	0,002		0,08028					
7	0,8	14,9	0,64	11,92	14,826	0,0055		0,08028					
8	3	91,1	1,76	45,24		0,0187		0,38833					
9	D	6	3	1,56	D1	91,1	3	24,616	D2	6	91,1	-1,86	
10		3	1,76			45,24	1,76			3	45,24		
11													
12													
13	a1	-1,19											
14	a0	15,78											
15													
16	IK	0,976											
17													

Fig. 1 The calculation of coefficients of linear dependence of the MLS in Excel

If we want to find out how close to the measured values a related line (a graph of a linear dependence) or a graph of a hyperbolic dependence is lying, it is necessary to calculate the correlation index I_K. As we know, for the correlation index it is valid that $I_K \in \langle 0,1 \rangle$ and a higher correlation value expresses a higher accuracy and dependence between a real and approximated function. In our case, in the cell B16 for a linear dependence $y = -1{,}19x + 15{,}78$ the correlation index $I_K = 0{,}976$ is calculated. When searching for a hyperbolic dependence $y = \frac{a_1}{x} + a_0$, the calculation procedure in Excel is similar, because the substitution $u = \frac{1}{x}$ leads us to a linear dependence $y = a_1 u + a_0$. We can see that for a searched hyperbolic dependence $y = \frac{0{,}188}{x} + 14{,}712$ is $I_K = 0{,}9635$ lower. A linear dependence thus approximates the measured values more accurately.

Conclusion

MS Excel is able to interpret obtained results graphically, to create graphs of various types in a very simple way. A graphical interpretation of the results enables students to self check if the graph of the searched dependence (function) really is close to the measured values. We can state that introduction of technical applications to the teaching of Applied Mathematics has its grounds. Students were motivated to be successful when solving tasks related to the environment of technical practice. The utilization of information technologies in the education process, in particular within the teaching of Applied Mathematics, significantly contributes to making students and teachers work easier and more effective.

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Assessed by: Assoc. Prof. PaedDr. Peter Beisetzer, PhD.

Contact Address:

Alena Vagaská, PaedDr. PhD.,
Katedra matematiky,informatiky a kybernetiky, Fakulta výrobných technológií výchovy TU v Košiciach so sídlom v Prešove, Bayerova 1, 080 01 Prešov, SR, tel. 00421917909564, e-mail: alena.vagaska@tuke.sk

Miroslav Gombár, Ing. PhD., Katedra výrobných technológií, Fakulta výrobných technológií TU v Košiciach so sídlom v Prešove, Štúrova 31, 080 01 Prešov, SR, e-mail: miroslav.gombar@tuke.sk

Ján Kmec, doc. Ing. CSc., Strojnícka fakulta, Technická univerzita v Košiciach, Katedra technológií a materiálov, Mäsiarska 74, 040 01 Košice, e-mail: jan.kmec@tuke.sk